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Meson-like Baryons and the Spin-Orbit Puzzle

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I describe a special class of meson-like Λ_Q excited states and present evidence supporting the similarity of their spin-independent spectra to those of mesons. I then examine spin-dependent forces in these baryons, showing that predicted effects of spin-orbit forces are small for them for the same reason they are small for the analogous mesons: a fortuitous cancellation between large spin-orbit forces due to one-gluon-exchange and equally large inverted spin-orbit forces due to Thomas precession in the confining potential. In addition to eliminating the baryon spin-orbit puzzle in these states, this solution provides a new perspective on spin-orbit forces in all baryons.

I. BACKGROUND

Confinement in mesons and baryons should be very similar since color dynamics is sensitive at large distances only to the net color charge of the interacting sources. Thus whether a quark is bound to an antiquark or to a diquark in a color $\bar{3}$ (as it must be for a baryon to be in an overall color singlet) should make no difference at large separation (or alternatively at high excitation). For similar reasons, the one-gluon-exchange forces in mesons and baryons are very closely related: pairwise forces in baryons have exactly half the strength of those in mesons (for identical spins and separations).

Given this close connection between meson and baryon dynamics and the success of the quark model in meson spectroscopy, it is surprising that there is still an unsettled qualitative problem in the quark model for baryons: the so-called "baryon spin-orbit puzzle" that baryon spin-orbit splittings appear to be much smaller than expected from their one-gluon-exchange matrix elements [1]. However, by this criterion the mesons would also have a spin-orbit problem. Meson spin-orbit splittings are also much smaller than expected from their one-gluon-exchange matrix elements [2,3], but mesons have no spin-orbit problem because the "normal" spin-orbit matrix element is largely cancelled by a strong "inverted" spin-orbit matrix element from Thomas precession in the confining potential.

The physics behind this cancellation has received support recently from analyses of heavy quarkonia, where both analytic techniques [4] and numerical studies using lattice QCD [5] have shown that the confining forces are spin-independent *apart* from the inevitable spin-orbit pseudoforce due to Thomas precession. Moreover, as has been known for more than ten years, the data on charmonia require an inverted spin-orbit matrix element from Thomas precession in the confining potential to cancel part of the strength of the OGE matrix element [2]. If the charm quark were sufficiently massive, its low-lying spectrum would be rigorously dominated by one gluon exchange, and one indeed observes that the Υ system is closer to this ideal. Conversely, as one moves from $c\bar{c}$ to lighter quarks, the $\ell = 1$ wave functions move farther out into the confining potential and the relative strength of the Thomas precession

term grows. It is thus very natural to expect a strong cancellation in light quark systems.

As shown in the original Isgur-Karl paper on the P-wave baryons [1], a very similar cancellation appears to occur at the two-body level in baryons. However, unlike mesons, baryons can also experience three-body spin-orbit forces [6] (*e.g.,* potentials proportional to $(\vec{S}_1 - \vec{S}_2) \cdot (\vec{r}_1 - \vec{r}_2) \times \vec{p}_3$ where $\vec{S}_i$, $\vec{r}_i$, and $\vec{p}_i$ are the spin, position, and momentum of quark i). The matrix elements of these three body spin-orbit forces are all calculated in Ref. [1], but no apparent cancellation amongst them is found. *I.e.,* the spin-orbit puzzle might more properly be called the "baryon three-body spin-orbit puzzle". In view of the facts that one could understand the smallness of spin-orbit forces in mesons and that the data also clearly called for small spin-orbit forces in baryons, the Isgur-Karl model anticipated a solution to the baryon three-body spin-orbit puzzle and *as a first approximation* discarded all spin-orbit forces. It was assumed that, as in mesons, a more precise and broadly applicable description would have to treat residual spin-orbit interactions. In the meantime, a possible solution to this problem has been suggested [7] in which relativistic effects enhance spin-spin over spin-orbit effects. This suggestion may prove to be correct, though it would then be an accident that in mesons a nonrelativistic solution presents itself. In this paper I identify a special class of baryons which exhibit a meson-like solution to the spin-orbit puzzle and lead to a new perspective on this old problem.

## II. A TOWER OF MESON-LIKE Λ_Q EXCITED STATES

A. Introduction

Consider a udQ baryon in which the ud quark pair is compact and Q is far from their center-of-mass, as shown in Fig. 1(b). As previously mentioned, the ud pair must be in a color $\bar{3}$, so the forces between it and Q are the same as those between an antiquark and Q. If the internal dynamics of the ud pair were independent of $\vec{\lambda}$, then each ud eigenstate would act as an extended quasi-antiquark with which Q could form a tower of meson-like

excited states. The simplest such *ud* pair, and the one which is the focus of this paper, has the *ud* pair in its isospin zero and spin zero ground state. I label these state Λ_{Q*}'s since in them only the Q relative coordinate is excited over the ground state $\Lambda_Q \frac{1}{2}^+$. We will see that the only spin-dependent forces in these states are the spin-orbit forces experienced by Q, so they are a natural choice for a system in which to investigate the baryon spin-orbit puzzle.
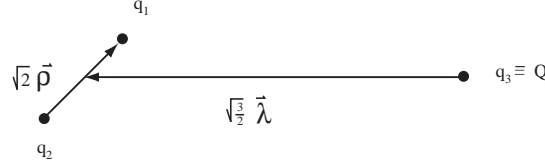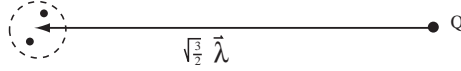
Fig. 1: (a) The relative coordinates $\vec{\rho} \equiv \sqrt{\frac{1}{2}}(\vec{r}_1 - \vec{r}_2)$ and $\vec{\lambda} \equiv \sqrt{\frac{1}{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$ of a $q_1 q_2 q_3$ baryon with $m_1 = m_2 = m$ and $m_3 = m_Q$. (b) A meson-like baryon configuration.

The idealization of a separation between $\vec{\rho}$ and $\vec{\lambda}$ dynamics (see Fig. 1(a)) is actually realized in the zeroth-order states of the Isgur-Karl (IK) model [1] where for $m_1 = m_2 = m$ and $m_3 = m_Q$

$$H = \frac{p_\rho^2}{2m} + \frac{p_\lambda^2}{2m_\lambda} + V_{si} + V_{sd} \tag{1}$$

with

$$m_\lambda \equiv \frac{3mm_Q}{2m + m_Q} \tag{2}$$

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and V_{si} and V_{sd} spin-independent and spin-dependent potentials. In the IK model one introduces an *artificial* harmonic term to define

$$V_{si} = \sum_{i<j} \frac{1}{2} k r_{ij}^2 + \Delta V_{si} \tag{3}$$

where

$$\Delta V_{si} \equiv V_{si} - \sum_{i<j} \frac{1}{2} k r_{ij}^2 \tag{4}$$

and then treats ΔV_{si} and V_{sd} as perturbations on the zeroth-order Hamiltonian

$$H_0 = \frac{p_\rho^2}{2m} + \frac{p_\lambda^2}{2m_\lambda} + \sum_{i<j} \frac{1}{2} k r_{ij}^2 \tag{5}$$

$$= \frac{p_\rho^2}{2m} + \frac{p_\lambda^2}{2m_\lambda} + \frac{3}{2} k \rho^2 + \frac{3}{2} k \lambda^2 \tag{6}$$

which separates as advertized with $\omega_\rho = \sqrt{3k/m}$ and $\omega_\lambda = \sqrt{3k/m_\lambda}$. (The choice of $\vec{\rho}$ and $\vec{\lambda}$ as variables was made historically so that in the SU(3) limit $m_s \to m$ the ρ and λ oscillators become degenerate with wave functions that are manifestly good representations of the permutation group S_3.) Note that by construction k is an auxillary parameter which can be chosen to minimize the importance of ΔV_{si}. In what follows I take

$$V_{si} = - \sum_{i<j} \frac{2\alpha_s}{3r_{ij}} + c + V_{conf} \tag{7}$$

where the first term is the color Coulomb potential between quarks in a baryon (to be contrasted with $-4\alpha_s/3r$ in a meson), c is a constant, and V_{conf} is the long-range confining potential. The zeroth-order eigenstates of (6) are states with spatial wave functions $\psi_{n_\rho \ell_\rho m_\rho}(\vec{\rho}) \psi_{n_\lambda \ell_\lambda m_\lambda}(\vec{\lambda})$ and with the various flavor and spin states allowed by the generalized Pauli principle (given that the quarks are in the totally antisymmetric color state C_A).

As stated earlier, I focus here on the isospin zero, light-quark-spin-zero states Λ_{Q^*} (i.e., those with flavor wave function

$$\phi_\Lambda = \sqrt{\frac{1}{2}}(ud - du)Q \tag{8}$$

and spin wave functions

$$\chi^{\rho}_+ = \sqrt{\frac{1}{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \tag{9}$$

$$\chi^{\rho}_- = \sqrt{\frac{1}{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \tag{10}$$

as defined in Ref. [1]) with the $\vec{\rho}$ variable in its ground state:

$$\psi_{Q^*_{n_\lambda \ell_\lambda m_\lambda}} = C_A \phi_\Lambda \chi^{\rho} \psi_{000}(\vec{\rho}\,)\psi_{n_\lambda \ell_\lambda m_\lambda}(\vec{\lambda}) \tag{11}$$

where $\psi_{n_v \ell_v m_v}(\vec{v})$ is the normalized harmonic oscillator wave function for the variable $\vec{v}$ with principal quantum number n_v and angular momentum quantum numbers (ℓ_v, m_v). Since $\vec{\lambda}$ is symmetric under $1 \leftrightarrow 2$ interchange, these states all have the $1 \leftrightarrow 2$ antisymmetry required by the generalized Pauli principle, and the tower of $\vec{\lambda}$ excited states Λ_{Q^*} stand in one-to-one correspondence with the states of a $\bar{\sigma}Q$ meson with potential $\frac{1}{2}(2k)r^2$ where $\vec{r} \equiv \vec{r}_{\bar{\sigma}} - \vec{r}_Q$ and $\bar{\sigma}$ is a ficticious antiquark with spin and isospin zero and mass $2m$. In the harmonic limit each of these towers of excited states have a spacing of $\sqrt{3k/m_\lambda} = \sqrt{2k/\mu_{\sigma Q}}$ where $\mu_{\sigma Q}$ is the $\bar{\sigma}Q$ reduced mass.

B. Beyond the Harmonic Approximation

As mentioned earlier, k is an auxillary parameter which may be chosen to minimize the perturbation ΔV_{si}. Since, consistent with the $1 \leftrightarrow 2$ symmetry of this system, Eq. (3) is trivially generalized to allow the r_{12} spring constant to be distinct from the r_{13} and r_{23} spring constants, the ρ and λ spring constants may be taken to be independent auxillary parameters k_ρ and k_λ. Somewhat less trivial is the fact that k_λ may be chosen independently for each value of ℓ_λ in the tower of λ excitations. The subtowers consisting of states of fixed ℓ_λ (but excitation labelled by n_λ) are all mutually orthogonal, so choosing k_λ to optimize the energy and wave function of $\psi_{0\ell_\lambda m_\lambda}(\vec{\lambda})$ is a good strategy for producing accurate orthonormalized eigenfunctions of the spin-independent Hamiltonian. (I optimize $n_\lambda = 0$ for each ℓ_λ since these are the phenomenologically most relevant states). Given this strategy, one never actually resorts to perturbation theory in ΔV_{si} for the $n_\lambda = 0$ states: their energies and

eigenfunctions are best determined variationally. The analogous strategy may obviously be employed for mesons.

In an elementary $\bar{\sigma}Q$ system with

$$V_{si} = -\frac{4\alpha_s}{3r} + c' + br \tag{12}$$

the expectation value of the spin-independent Hamiltonian in $\psi_{0\ell m}(r) \sim Y_{\ell m}(\theta\phi)r^\ell e^{-\frac{1}{2}\beta^2 r^2}$ is

$$E_{0\ell}(\beta) = \left(\frac{2\ell+3}{2}\right)\frac{\beta^2}{2\mu_{\sigma Q}} - \frac{2^{\ell+3}\ell!\alpha_s\beta}{3(2\ell+1)!!\sqrt{\pi}} + c' + \frac{2^{\ell+1}(\ell+1)!b}{(2\ell+1)!!\sqrt{\pi}\beta} \quad, \tag{13}$$

where $(2\ell+1)!! \equiv (2\ell+1)(2\ell-1)(2\ell-3)\cdots 3\cdot 1$. Table I shows how this spectrum varies with $\mu_{\sigma Q}$. (Throughout this paper I use the "standard" parameters $m_u = m_d \equiv m = 0.33$ GeV, $m_s = 0.55$ GeV, $m_c = 1.82$ GeV, $m_b = 5.20$ GeV, and $b = 0.18$ GeV^2, with $\alpha_s = 0.6$ since we are considering large-distance dominated systems.) In a pure Coulomb potential, the spacings between energy levels would grow like $\mu_{\sigma Q}$, while in a pure linear potential they would decrease like $\mu_{\sigma Q}^{-1/3}$. The latter behaviour can be seen at large ℓ, but at low ℓ the admixture of Coulomb potential relevant to light quark spectroscopy leads to even more slowly varying splittings, as observed in nature (see below).

TABLE I. The variationally determined elementary meson spectrum (in GeV) as a function of the reduced mass $\mu_{\sigma Q}$.

spectral splitting	$\mu_{\sigma Q} = m/2$	$\mu_{\sigma Q} = m$	$\mu_{\sigma Q} = 2m$
$E_{01} - E_{00}$	0.59	0.53	0.51
$E_{02} - E_{01}$	0.45	0.38	0.33
$E_{03} - E_{02}$	0.39	0.32	0.27
$E_{04} - E_{03}$	0.35	0.28	0.23

We expect the analogous tower of Λ_{Q^*} baryons to have the same spectrum for sufficiently large ℓ. I will now demonstrate that this is the case. The relevant baryon wave functions are those of Eq. (11) with

$$\psi_{000}(\vec{\rho}\,) = \frac{\alpha_\rho^{3/2}}{\pi^{3/4}} e^{-\frac{1}{2}\alpha_\rho^2\rho^2} \tag{14}$$

and

$$\psi_{0\ell\ell}(\vec{\lambda}) = \frac{\alpha_\lambda^{\ell+3/2}\lambda_+^\ell}{\pi^{3/4}\sqrt{\ell!}} e^{-\frac{1}{2}\alpha_\lambda^2\lambda^2} \tag{15}$$

where $\lambda_\pm \equiv \lambda_1 \pm i\lambda_2$. The kinetic energy term analogous to the first term in Eq. (13) is thus

$$\frac{3\alpha_\rho^2}{4m} + \left(\frac{2\ell+3}{2}\right)\frac{\alpha_\lambda^2}{2m_\lambda} \quad . \tag{16}$$

More interesting are the Coulomb terms analogous to the second term of Eq. (13) arising from Eq. (7). The $-2\alpha_s/3r_{12}$ term is straightforward, but the $-2\alpha_s/3r_{13}$ and $-2\alpha_s/3r_{23}$ terms have an interesting wrinkle: Gauss' Law. In averaging $\vec{\rho}$ over a spherical shell around the ud center-of-mass, only shells that fall between that center-of-mass and Q will lead to an electric field at Q. This electric field will be that of the charge of the shell concentrated at the center of mass, and will have the corresponding potential energy. A shell that falls outside Q will produce no electric field and a constant potential corresponding to the potential that a distant charge would have experienced just as it crossed the shell. Thus in the special class of states we are considering here, the color Coulomb potential takes on the effective form

$$V^{eff}_{Coulomb}(\vec{\rho}, \vec{\lambda}) = V^{eff}_\rho(\rho) + V^{eff}_\lambda(\rho, \lambda) \tag{17}$$

where

$$V^{eff}_\rho(\rho) = -\frac{2\alpha_s}{3(\sqrt{2}\rho)} \tag{18}$$

and

$$V^{eff}_\lambda(\rho, \lambda) = -\frac{4\alpha_s}{3}\left[\frac{1}{(\sqrt{\frac{3}{2}}\lambda)}\theta(\lambda - \frac{\rho}{\sqrt{3}}) + \frac{1}{(\sqrt{\frac{1}{2}}\rho)}\theta(\frac{\rho}{\sqrt{3}} - \lambda)\right] \quad . \tag{19}$$

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We may associate V_ρ^{eff} with a color electric field internal to the ud pair and directed along $\vec{\rho}$ and V_λ^{eff} with the color electric field described above and directed along $\vec{\lambda}$.

After averaging over ρ in the wave function $\psi_{000}(\vec{\rho}\,)$ one obtains a λ-dependent effective potential

$$V_{Coulomb}^{eff}(\lambda) = -\frac{4\alpha_s\alpha_\rho}{3\sqrt{2\pi}} - \frac{4\sqrt{2}\alpha_s Q_{\alpha_\rho}(\sqrt{\frac{3}{2}}\lambda)}{3\sqrt{3}\lambda} - \frac{16\alpha_s\alpha_\rho}{3\sqrt{2\pi}}e^{-3\alpha_\rho^2\lambda^2} \tag{20}$$

$$= -\frac{4\alpha_s\alpha_\rho}{3\sqrt{2\pi}} - \frac{4\sqrt{2}\alpha_s \; erf(\sqrt{3}\alpha_\rho\lambda)}{3\sqrt{3}\lambda} \tag{21}$$

where

$$Q_{\alpha_\rho}(\sqrt{\frac{3}{2}}\lambda) \equiv \frac{\alpha_\rho^3}{\pi^{3/2}}\int_0^{\sqrt{3}\lambda} d^3\rho \; e^{-\alpha_\rho^2\rho^2} \tag{22}$$

is the "charge" inside the radius $\sqrt{\frac{3}{2}} \lambda$ and $erf(z) \equiv \frac{1}{\sqrt{\pi}}\int_{-z}^{z} dx e^{-x^2}$. Notice that

$$E_{Coulomb}^{eff}(\lambda) \equiv -\frac{dV_{Coulomb}^{eff}}{d(\sqrt{\frac{3}{2}}\lambda)} = -\frac{4\alpha_s Q_{\alpha_\rho}(\sqrt{\frac{3}{2}}\lambda)}{3(\sqrt{\frac{3}{2}}\lambda)^2} \tag{23}$$

as required, and that the energy associated with the Coulomb potentials is the expectation value of $V_{Coulomb}^{eff}(\lambda)$ in the wave function $\psi_{0\ell\ell}(\vec{\lambda})$. Due to the appearance of $Q_{\alpha_\rho}(\sqrt{\frac{3}{2}}\lambda)$ this energy cannot be displayed in closed form, and variational solutions must be found numerically. However, since $\langle\lambda^2\rangle^{1/2} = \sqrt{\ell + \frac{3}{2}} \; /\alpha_\lambda$ and since the α_λ which minimizes the energy decreases as ℓ increases, the Coulomb energy quickly "heals" to the meson-like value

$$-\frac{4\alpha_s\alpha_\rho}{3\sqrt{2\pi}} - \frac{2^{\ell+3}\ell!\alpha_s(\sqrt{\frac{2}{3}}\alpha_\lambda)}{3(2\ell+1)!!\sqrt{\pi}} \tag{24}$$

where the first term is the ud energy and $\sqrt{\frac{2}{3}}\alpha_\lambda$ is the appropriate analog of β since $\sqrt{\frac{3}{2}}\lambda$ corresponds to r. In practice this "healing" occurs very rapidly: in the Λ_{s*} baryons we will consider below, the Coulomb energy differs from its meson-like value by only 5% in $\ell = 1$ and 2% in $\ell = 2$.

Finally, we consider the confining potential. From the preceding discussion we know that the color Coulomb field in the special class of states we are considering here has two

components: one of magnitude $-2\alpha_s/3(\sqrt{2}\rho)^2$ internal to the ud pair which is directed along $\vec{\rho}$, and one of asymptotic magnitude $-4\alpha_s/3(\sqrt{\frac{3}{2}}\lambda)^2$ between the ud pair and Q which is directed along $\vec{\lambda}$. Since confinement evolves out of the color electric field at large distances, it is natural to assume, in analogy to the standard meson hypothesis encapsulated in Eq. (12), that in these states

$$V_{conf}^{eff}(\rho, \lambda) = \frac{1}{2}b(\sqrt{2}\rho) + b(\sqrt{\frac{3}{2}}\lambda) \tag{25}$$

up to an overall constant. Since the second term is the analog of the meson confining potential br, the demonstration that at sufficiently large ℓ our tower of Λ_{Q^*} baryons will have the same spectrum as the $\bar{\sigma}Q$ mesons is complete.

In summary, I have shown in this Section that in the variational wave functions (11), and *a fortiori* in lowest order perturbation theory in ΔV_{si} of Eq. (4), a tower of Λ_{Q^*} baryons with the ud pair in $\ell_\rho = 0$ and spin zero exists which will be analogous to the spectrum of a ficticious meson $\bar{\sigma}Q$ containing a scalar antiquark of mass $2m$. Gauss' Law has produced a situation that is only slightly more complicated than the idealized harmonic limit wherein the $\vec{\rho}$ and $\vec{\lambda}$ variables completely separate: they have become coupled only through the effect of $\psi_{000}(\rho)$ on the effective charge $Q_{\alpha_\rho}(\sqrt{\frac{3}{2}}\lambda)$. Physically this means that the size of the ud pair's wave function is not fixed, but rather that it grows to an asymptotic value as ℓ_λ increases.

C. An Empirical Meson-Baryon Correspondence

Though there are no mesons with scalar antiquarks $\bar{\sigma}$, the spin-independent quark model spectra of such mesons would be the same as those of ordinary $\bar{q}Q$ mesons with $m_q = m_\sigma$. Moreover, Figs. 2 and 3 show what Table I has led us to anticipate: the spectra of mesons are slowly-varying functions of their reduced masses. Thus we may reasonably look for a correspondence between the spectrum of any flavor of mesons with reduced mass of the order of m with the select tower of Λ_{Q^*} baryons associated with any heavy quark Q. The most

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extensive data exists for the K^* and the ordinary Λ^* (i.e., Λ_s^*) states, as it happens, and this correspondence is shown for the "fully stretched" total angular momentum states of these systems in Fig. 4. I first note that a comparison of the observed splittings in the K^* system with those of Table I indicates that the framework adopted here is in fact quite reasonable. Since the $K^* - \Lambda_{s*}$ correspondence should be best for high ℓ, I have aligned the highest ℓ established for both spectra, namely $\ell = 3$ corresponding to a 4^+ K^* and a $\frac{7}{2}^-$ Λ_{s*}. The excellent correspondence at high ℓ provides a detailed view in the context of the quark model of the well-known relation between the slopes of meson and baryon Regge trajectories. That the $\ell = 0 \to \ell = 1$ and $\ell = 1 \to \ell = 2$ splittings are smaller in the Λ_{s*}'s than in the K^*'s is what we expect from the analysis of the previous Section: for low ℓ the Coulomb potential is weakened since the ud color charge is distributed in the wave function $\psi_{000}(\vec{\rho}\,)$. Figure 4 therefore also shows the spectrum of a ficticious $\tilde{\Lambda}_{s*}$ system obtained from the experimental Λ_{s*} spectrum by adding to it the perturbation that would occur if the spatial extension of the ud pair were set to zero, demonstrating that even the low ℓ behavior of the K^* and Λ_{s*} systems are related as expected.
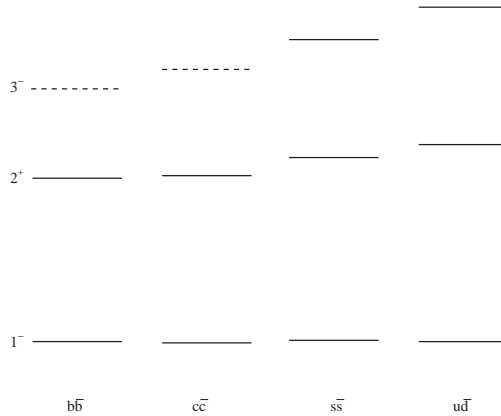

Fig. 2: The orbital excitation spectra of the quarkonia $Q\bar{Q}$ as a function of m_Q; the 1^- S-waves have been aligned to display the splittings to the P-wave 2^+ and D-wave 3^- states. In $b\bar{b}$ and $c\bar{c}$, the D-wave positions shown as dashed lines are predictions of reliable heavy quarkonium calculations. The spectra are shown to scale, which may conveniently be calibrated from the $\chi_{c2} - \psi$ splitting of 459 MeV.
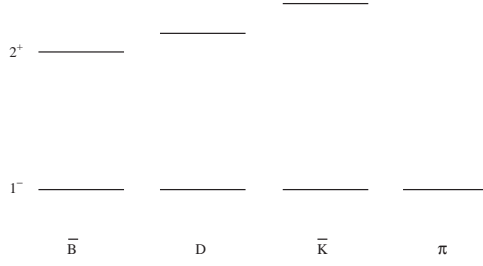
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Fig. 3: The orbital excitation spectra of heavy-light mesons as a function of m_Q; the 1^- S-waves have been aligned to display the splittings to the P-wave 2^+ states. The spectra are shown to scale, which may conveniently be calibrated from the $D_2^* - D^*$ splitting of 452 MeV.
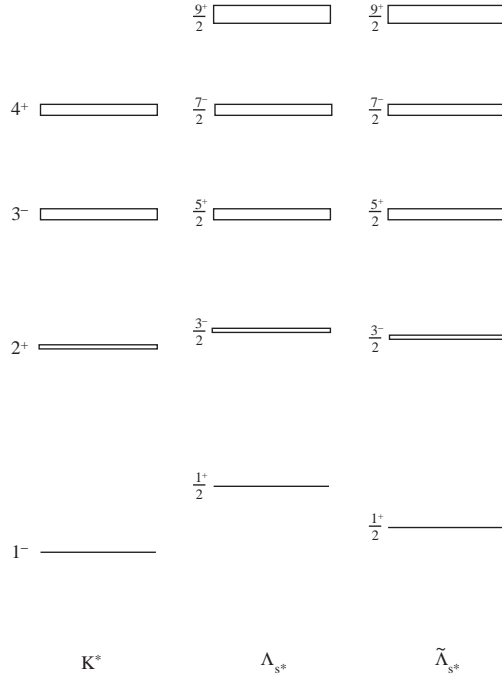

Fig. 4: Comparison of the spectra of well-established K^* mesons and Λ_{s*} baryons. The boxes shown represent the experimental uncertainties in the masses of states. The $\ell = 3$ states have been aligned as described in the text. The third spectrum is that of a fictitious $\tilde{\Lambda}_{s*}$ baryon with the spatial distribution of the ud pair set to zero. The spectra are shown to scale, which may conveniently be calibrated from the $K_2^* - K^*$ splitting of 534 MeV.

III. SPIN-ORBIT FORCES IN Λ_{Q^*} BARYONS

Given the remarkable correspondence between the spin-independent spectra of mesons and the Λ_{Q^*} baryons, this special tower of states seems ideal for investigating the baryon spin-orbit puzzle. Since the color dynamics of these two systems are so similar, and since there is no meson spin-orbit puzzle, one would naively expect to find no problem with spin-orbit forces in the Λ_{Q^*} states. This is in fact what I find.

A. A Review of Meson Spin-Orbit Forces

I begin with a review of meson spin-orbit forces. In a "real" $\bar{q}Q$ meson with two spin-1/2 particles, the spin-dependent potential is

$$V_{sd}^{\bar{q}Q} = V_{ss}^{\bar{q}Q} + V_{so}^{\bar{q}Q} \tag{26}$$

where $V_{ss}^{\bar{q}Q}$ is the usual spin-spin interaction (consisting of the Fermi contact term and the tensor interaction) and

$$V_{so}^{\bar{q}Q} = \frac{4\alpha_s}{3\mu_{\bar{q}Q}r^3}\vec{L}\cdot\left(\frac{\vec{S}_Q}{m_Q} + \frac{\vec{S}_{\bar{q}}}{m_q}\right) - \left(\frac{2\alpha_s}{3r^3} + \frac{b}{2r}\right)\vec{L}\cdot\left(\frac{\vec{S}_Q}{m_Q^2} + \frac{\vec{S}_{\bar{q}}}{m_q^2}\right) \quad . \tag{27}$$

Here the first term is the dynamic spin-orbit interaction arising from the interaction of the color magnetic moments $\vec{S}_Q/m_Q$ and $\vec{S}_{\bar{q}}/m_q$ with the color magnetic fields generated by the motion of the other quark and the second term is the kinematic spin-orbit effect arising from Thomas precession in the central spin-independent potential. One can equivalently write

$$V_{so}^{\bar{q}Q} = \frac{2\alpha_s}{3r^3}\vec{L}\cdot\left[\vec{S}_Q\left(\frac{1}{m_Q^2} + \frac{2}{m_Qm_q}\right) + \vec{S}_{\bar{q}}\left(\frac{1}{m_q^2} + \frac{2}{m_Qm_q}\right)\right] - \frac{b}{2r}\vec{L}\cdot\left[\frac{\vec{S}_Q}{m_Q^2} + \frac{\vec{S}_{\bar{q}}}{m_q^2}\right] \tag{28}$$

in which form the matrix elements of $1/r^3$ and $1/r$ are separated, corresponding to the complete one-gluon-exchange component and the complete confinement component of the spin-orbit interaction, respectively. In the $m_Q = m_q = m$ isovector meson sector

$$V_{so}^{\bar{q}Q} \to V_{so}^{\bar{d}u} = \vec{L}\cdot\vec{S}\left[\frac{2\alpha_s}{m^2r^3} - \frac{b}{2m^2r}\right] \tag{29}$$

where $\vec{S} = \vec{S}_Q + \vec{S}_{\bar{q}} = \vec{S}_u + \vec{S}_{\bar{d}}$. When evaluated in the $\ell = 1$ wave functions of the spin-independent potential (12), the one-gluon-exchange component has a matrix element of $+180$ MeV, while the Thomas precession component has a matrix element of -170 MeV. (These matrix elements are multiplied by $\langle \vec{L} \cdot \vec{S} \rangle$ which is $+1$, -1, -2, and 0 in the a_2, a_1, a_0, and b_1, respectively.) The total matrix element of Eq. (29) can be extracted from the mixture of contact, tensor, and spin-orbit terms contributing to the experimental P-wave masses from the combination $\frac{5}{12}a_2 - \frac{1}{4}a_1 - \frac{1}{6}a_0$ and is -3 ± 20 MeV. We see that the mesons would have a serious spin-orbit problem if Thomas precession in the confining potential were ignored, but that the sum of the one-gluon-exchange spin-orbit forces and confining Thomas precession spin-orbit forces leads to a very small net spin-orbit splitting as observed.

In heavier quarkonia $Q\bar{Q}$, as m_Q increases $\langle 1/m_Q^2 r^3 \rangle$ decreases (in a linear potential it would decrease exactly like m_Q^{-1}), but the ratio of the matrix element of $1/r$ to that of $1/r^3$ also decreases (like $m^{-2/3}$ in a linear potential) so that very heavy quarkonia are totally dominated by one-gluon-exchange spin-orbit forces. As previously mentioned, this decreasing importance of Thomas precession in the confining potential is observed in the χ_c and χ_b states.

In heavy-light mesons $\bar{d}Q$ in the heavy quark limit $m_Q \to \infty$,

$$V_{so}^{\bar{q}Q} \to V_{so}^{\bar{d}Q}|_{m_Q \to \infty} = \vec{L} \cdot \vec{S}_{\bar{d}} \left[\frac{2\alpha_s}{3m^2 r^3} - \frac{b}{2m^2 r} \right] \quad . \tag{30}$$

It is this interaction which determines the splitting between the heavy quark spin multiplets [8] with $s_\ell^{\pi_\ell} = 3/2^+$ and $s_\ell^{\pi_\ell} = 1/2^+$ associated with the $\ell = 1$ excitations of the $\bar{d}Q$ system in the quark model. Since the wave function parameter β (see Eq. (13)) increases by only about 25% between $\bar{d}u$ and $\bar{d}Q$, it is to be expected [3] from comparing Eqs. (30) and (29) that these two multiplets will be inverted. Unfortunately, no $s_\ell^{\pi_\ell} = 1/2^+$ charm or beauty mesons are known, so this expectation is untested experimentally [9]. The spin-orbit splitting *inside* the $s_\ell^{\pi_\ell} = 3/2^+$ multiplet is controlled in leading order in $1/m_Q$ by the pure one-gluon-exchange operator

$$\frac{4\alpha_s}{3mm_Q r^3}\vec{L}\cdot\vec{S}_Q \quad . \tag{31}$$

This operator produces a predicted splitting between D_2^* and D_1 of the charm $s_\ell^{\pi\ell} = 3/2^+$ spin multiplet of 50 MeV, comparable to the observed splitting of 45 MeV. Spin-spin interactions can also contribute to this splitting at this order in $1/m_Q$, but their calculated effect is very small ($\simeq 5$ MeV).

In summary, mesons not only have no spin-orbit problem, but their splittings are reasonably well described by the standard spin-orbit interaction (27).

B. The Λ_{Q^*} Baryons: Theory

The meson-like tower of Λ_{Q^*} baryons is actually simpler than the analogous $\bar{d}Q$ mesons because with only the spin $\vec{S}_Q$ of the heavy quark active, spin-orbit forces are the only spin-dependent forces in first order perturbation theory. This is because in the Λ_{Q^*} states the ud spin-zero wave function factorizes from the dynamical parts of the wave function and $\langle\vec{S}_1\rangle$ and $\langle\vec{S}_2\rangle$ are zero, leaving as the only operative spin-dependent interaction

$$V_{so}^{\Lambda_{Q^*}} = \left[\frac{1}{m_Q\mu_{\sigma Q}(\sqrt{\frac{3}{2}}\lambda)}\frac{dV_{Coulomb}^{eff}}{d(\sqrt{\frac{3}{2}}\lambda)} - \frac{1}{m_Q^2}\left(\frac{1}{2(\sqrt{\frac{3}{2}}\lambda)}\frac{dV_{Coulomb}^{eff}}{d(\sqrt{\frac{3}{2}}\lambda)} + \frac{b}{2(\sqrt{\frac{3}{2}}\lambda)}\right)\right]\vec{L}_\lambda\cdot\vec{S}_Q \quad , \tag{32}$$

where the first and second terms, in an obvious notation, are $V_{so(dynamic)}^{\Lambda_{Q^*}}$ and $V_{so(Thomas)}^{\Lambda_{Q^*}}$. In the heavy quark limit [8] $m_Q \to \infty$ this interaction becomes in leading order in $1/m_Q$ simply

$$\frac{1}{m_Q\mu_{\sigma Q}(\sqrt{\frac{3}{2}}\lambda)}\frac{dV_{Coulomb}^{eff}}{d(\sqrt{\frac{3}{2}}\lambda)}\vec{L}_\lambda\cdot\vec{S}_Q \quad , \tag{33}$$

the operator analogous to that responsible for the $D_2^* - D_1$ splitting. In the very crude approximation of taking the $\ell = 1$ D and Λ_{c^*} wave functions to be identical, approximating $dV_{Coulomb}^{eff}/d(\sqrt{\frac{3}{2}}\lambda)$ by $4\alpha_s/3(\sqrt{\frac{3}{2}}\lambda)^2$, working to leading order in $1/m_c$, and ignoring spin-spin interactions in the $D_2^* - D_1$ splitting, the $D^* - \Lambda_{Q^*}$ analogy would lead to the result

$$m_{\Lambda_c\frac{3}{2}^-} - m_{\Lambda_c\frac{1}{2}^-} \simeq \frac{1}{2}(m_{D_2^*} - m_{D_1}) \simeq 20 \text{ MeV} \tag{34}$$

in reasonable agreement with the observed splitting of 30 MeV [10]. If we go further and use $1/m_Q$ scaling down to m_s, the observed $\Lambda_c \frac{3}{2}^- - \Lambda_c \frac{1}{2}^-$ splitting would lead to $\Lambda \frac{3}{2}^- - \Lambda \frac{1}{2}^- \simeq 110$ MeV, in unreasonably good agreement with the observed $\Lambda(1520)\frac{3}{2}^- - \Lambda(1405)\frac{1}{2}^-$ splitting of 115 MeV. I will show below that these crude estimates are not that misleading, but from them one can already begin to anticipate the announced conclusion that there will also be no spin-orbit problem in the Λ_{Q^*} baryons.

This conclusion appears to be inconsistent with that of Ref. [1] where two-body spin-orbit forces invited a meson-like cancellation, but three-body spin-orbit forces did not. We will see in what follows that the inconsistency is only apparent: meson-like behaviour of the Λ_{Q^*} states implies that the three-body spin-orbit forces have a structure which leads to an overall "quasi-two-body" spin-orbit force between Q and the "quasi-antiquark" $\bar{\sigma}$ made of u and d.

In the picture I have adopted here, in which the ud pair is treated as an extended scalar antiquark $\bar{\sigma}$ of mass $2m$, the meson-like character of spin-orbit forces seems obvious. However, given its importance, we now examine this conclusion very carefully from a three-body perspective. The essential issues are all present in the simplified case where spin-dependent forces perturb zeroth-order harmonic oscillator states (11), and so it is this case I will discuss explicitly. In the absence of degeneracies in the spectrum, V_{sd} will perturb the energy of these Λ_{Q^*} states by

$$\Delta E_{n_\lambda \ell_\lambda} = \langle \psi_{Q^*_{n_\lambda \ell_\lambda \ell_\lambda}} | V_{sd} | \psi_{Q^*_{n_\lambda \ell_\lambda \ell_\lambda}} \rangle \ . \tag{35}$$

First consider a term of V_{sd} of the form $\vec{S}_1 \cdot \vec{V}$ where $\vec{V}$ is a vector operator formed from the other variables of the three body system. Since, $\langle \chi^\rho_\pm | \vec{S}_1 | \chi^\rho_\pm \rangle = 0$, such a term cannot contribute to $\Delta E_{n_\lambda \ell_\lambda}$. Similarly, $\vec{S}_2 \cdot \vec{V}$ terms do not contribute, so as described above only $\vec{S}_Q \cdot \vec{V}$ - type terms with $\vec{V}$ independent of $\vec{S}_1$ and $\vec{S}_2$, i.e., spin-orbit interactions of Q, can contribute. The one-gluon-exchange part of these interactions is

$$V_{so}^{oge} = \sum_{i=1,2} \frac{2\alpha_s}{3r_{iQ}^3} \frac{\vec{S}_Q}{m_Q} \cdot (\vec{r}_{iQ} \times [\frac{\vec{p}_i}{m} - \frac{\vec{p}_Q}{m_Q}] + \frac{1}{2m_Q} \vec{r}_{iQ} \times \vec{p}_Q) \tag{36}$$

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where the first and second terms are its dynamic and Thomas precession pieces, respectively.

I begin with the Thomas precession piece $V_{so(Thomas)}^{oge}$. The Λ_{Q^*} states of definite jm are of the form $|\Lambda_{Q_{jm}^*}\rangle = C_{\ell_\lambda m_\lambda \sigma}^{jm}|\psi_{Q_{n_\lambda \ell_\lambda m_\lambda}^*} Q_\sigma\rangle$ where $\sigma = \pm$ and the $C_{\ell_\lambda m_\lambda \sigma}$ are simple Clebsch-Gordan coefficients. In these states the expectation value of $V_{so(Thomas)}^{oge}$ is

$$\langle\Lambda_{Q_{jm}^*}|V_{so(Thomas)}^{oge}|\Lambda_{Q_{jm}^*}\rangle = C_{\ell_\lambda m_\lambda' \sigma'}^{jm\,*}C_{\ell_\lambda m_\lambda \sigma}^{jm}\sum_{i=1,2}\langle\sigma'|\int d^3\lambda\int d^3\rho$$
$$\psi_{n_\lambda \ell_\lambda m_\lambda'}^*(\vec{\lambda})\psi_{000}^*(\vec{\rho}\,)\frac{\alpha_s \vec{S}_Q \cdot \vec{r}_{iQ} \times \vec{p}_Q}{3m_Q^2 r_{iQ}^3}\psi_{000}(\vec{\rho}\,)\psi_{n_\lambda \ell_\lambda m_\lambda}(\vec{\lambda})|\sigma\rangle \quad (37)$$
$$= C_{\ell_\lambda m_\lambda \sigma'}^{jm\,*}C_{\ell_\lambda m_\lambda \sigma}^{jm}\sum_{i=1,2}\langle\sigma'|\int d^3\lambda$$
$$\psi_{n_\lambda \ell_\lambda m_\lambda}^*(\vec{\lambda})\frac{\vec{S}_Q \cdot \vec{E}_{iQ}(\vec{\lambda}) \times \vec{p}_Q}{2m_Q^2}\psi_{n_\lambda \ell_\lambda m_\lambda}(\vec{\lambda})|\sigma\rangle \quad (38)$$

since $\vec{S}_Q$ and $\vec{p}_Q = -\sqrt{\frac{2}{3}}\vec{p}_\lambda$ are independent of $\vec{\rho}$ [11], where

$$\vec{E}_{iQ}(\vec{\lambda}) = \int d^3\rho\,|\psi_{000}(\vec{\rho}\,)|^2\frac{2\alpha_s \vec{r}_{iQ}}{3r_{iQ}^3} \quad (39)$$

is the color electric field at the position of Q due to the quark i in the spherically symmetric wave function $\psi_{000}(\vec{\rho}\,)$. Thus

$$\langle\Lambda_{Q_{jm}^*}|V_{so(Thomas)}^{oge}|\Lambda_{Q_{jm}^*}\rangle = \langle\Lambda_{Q_{jm}^*}|V_{so(Thomas)}^{oge\,\Lambda_{Q^*}}|\Lambda_{Q_{jm}^*}\rangle \quad (40)$$

where

$$V_{so(Thomas)}^{oge\,\Lambda_{Q^*}} = -\frac{1}{2m_Q^2(\sqrt{\frac{3}{2}}\lambda)}\frac{dV_{Coulomb}^{eff}}{d(\sqrt{\frac{3}{2}}\lambda)}\vec{L}_\lambda \cdot \vec{S}_Q \quad (41)$$

as advertized in Eq. (32). The dynamical piece is somewhat more complex. Since, for example,

$$\vec{r}_{1Q} \times [\frac{\vec{p}_1}{m} - \frac{\vec{p}_Q}{m}] = (\sqrt{\frac{3}{2}}\vec{\lambda} + \sqrt{\frac{1}{2}}\vec{\rho}) \times (\sqrt{\frac{3}{2}}\frac{\vec{p}_\lambda}{m_\lambda} + \sqrt{\frac{1}{2}}\frac{\vec{p}_\rho}{m})$$
$$= \frac{3}{2m_\lambda}\vec{L}_\lambda + \frac{1}{2m}\vec{L}_\rho + \frac{\sqrt{3}}{2m_\lambda}\vec{\rho} \times \vec{p}_\lambda + \frac{\sqrt{3}}{2m}\vec{\lambda} \times \vec{p}_\rho \quad , \quad (42)$$

we must consider the expectation value analogous to that shown above with $\vec{r}_{iQ} \times \vec{p}_Q$ replaced by each of these four operators. Consider first $\vec{L}_\rho$: since $\vec{L}_\rho\psi_{000}(\vec{\rho}\,) = 0$, it vanishes

immediately. Next consider $\vec{\rho} \times \vec{p}_\lambda$. Since $\vec{p}_\lambda$ can be removed from the $\vec{\rho}$ integration, and since this integration can be organized in annular regions around the direction of $\sqrt{\frac{3}{2}}\vec{\lambda}$ which hold $|\vec{r}_{1Q}|$ fixed, $\int d^3\rho \, |\psi_{000}(\vec{\rho})|^2 \vec{\rho}/r_{1Q}^3 = 0$. Similarly, since $\vec{p}_\rho$ operating on $\psi_{000}(\rho)$ gives a result proportional to $\vec{\rho}\psi_{000}(\rho)$, the $\vec{\lambda} \times \vec{p}_\rho$ term vanishes. Thus the net effect of the dynamic term is carried by the $3\vec{L}_\lambda/2m_\lambda$ term (which is identical for $i = 1$ and 2). Using the preceding, one easily obtains

$$V_{so(dynamic)}^{oge \; \Lambda_{Q^*}} = \frac{1}{m_Q \mu_{\sigma Q}(\sqrt{\frac{3}{2}}\lambda)} \frac{dV_{Coulomb}^{eff}}{d(\sqrt{\frac{3}{2}}\lambda)} \vec{L}_\lambda \cdot \vec{S}_Q \tag{43}$$

as in Eq. (32). This completes our check that in the Λ_{Q^*} states one-gluon-exchange spin-orbit forces are purely meson-like.

In the absence of a microscopic picture of confinement, I do not know how to improve on the argument made earlier that since confinement arises out of the color electric field, Thomas precession in the confining potential will be that induced by a confining force directed along $\vec{\lambda}$, in the same direction as $\vec{E}_Q$ in Eq. (39). It should be noted that any microscopically two-body model for confinement of the form

$$V_{conf} = \sum_{i<j} V_{conf}^{ij}(r_{ij}) \tag{44}$$

will lead to this result since then the force on Q from particle 1 will be

$$\vec{F}_{1Q} = -\frac{1}{r_{1Q}} \frac{dV_{conf}^{1Q}}{dr_{1Q}} \vec{r}_{1Q} \;\; . \tag{45}$$

With, $\vec{r}_{1Q} = \sqrt{\frac{3}{2}}\vec{\lambda} + \sqrt{\frac{1}{2}}\vec{\rho}$, in a Λ_{Q^*} state only the $\sqrt{\frac{3}{2}}\vec{\lambda}$ term survives since the $\vec{\rho}$ integration can be done over annular regions of fixed $|\vec{r}_{1Q}|$. (The Coulomb potential is thus only special among two-body forces in that its dependence on λ is dictated by Gauss' Law, not that it leads to a force directed along $\vec{\lambda}$.) Given its implications for the relationship between the conclusions drawn here about spin-orbit forces and those in the literature, it is also useful to explicitly consider the case of "harmonic confinement" in which $V_{conf}^{ij} = \frac{1}{2}kr_{ij}^2$. In this case the total confining force on Q is always $\sqrt{6}k\vec{\lambda}$ *independent of the state of the $\vec{\rho}$ variable*, so the spin-orbit forces on Q from Thomas precession in a harmonic potential are *always purely quasi-two-body*.

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To summarize, we have found and confirmed that, as expected, the Λ_{Q*} baryons have no spin-orbit puzzle, and in particular that they have only meson-like quasi-two-body spin-orbit forces. This conclusion implies that the three-body spin-orbit forces described in the literature conspire in the case of Λ_{Q*} baryons to produce quasi-two-body behaviour. (One may readily check using the matrix elements provided in Ref. [1] that this is true for $\ell_\lambda = 1$.) We will discuss this new perspective on the baryon spin-orbit puzzle below.

C. The Λ_{Q*} Baryons: Phenomenology

The preceding discussion of spin-orbit forces in Λ_{Q*} baryons is important conceptually in defining a tower of baryon excitations with no spin-orbit problem and in creating a new perspective on the spin-orbit puzzle. However, it does not have much useful predictive power. In the Λ_{s*}'s, where there is substantial data, the two-body spin-orbit force is (as in the K^*'s) the ill-determined difference of a large positive dynamical spin-orbit term and a large negative Thomas precession term, while in the heavy quark sectors Λ_{c*} and Λ_{b*} the simple dynamical term dominates so that the predictions are reasonably reliable, but there is little data available. The calculations themselves are straightforward, requiring only the matrix elements of $V_{so}^{\Lambda_{Q*}}$ of Eq. (32) in the wave functions $\psi_{0\ell_\lambda\ell_\lambda}$ of Eq. (15). It should be noted that the required matrix elements (especially those of $1/r^3$) are not very accurately determined by these harmonic oscillator wave functions. This accuracy (which is typically about 25% in low-lying states) could easily be improved with better wave functions, the most significant qualitative aspect of such an improvement program being an increase of the $1/r^3$ relative to the $1/r$ matrix elements. However, the value of such an improvement is dubious: based on the expectation value of E/m, the nonrelativistic framework of this entire discussion can only be expected to be accurate at the $\pm 50\%$ level. I therefore present in Table II the separated dynamical and Thomas precession spin-orbit contributions to emphasize not the numerical results, but rather the cancellation that is at the heart of the solution of the spin-orbit puzzle. (Note that while the matrix elements of $1/r^3$ and $1/r$ are decreasing with

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ℓ, since $\langle \vec{L}_\lambda \cdot \vec{S}_Q \rangle$ grows, both contributions remain substantial through $\ell = 4$.) By assigning theoretical errors of $\pm 50\%$ to each contribution, the *quantitative* reliability of the various predicted splittings can also be assessed.

TABLE II.   The predicted spin-orbit splittings in Λ_{Q^*} baryons for $\ell =$1, 2, 3, and 4 (in MeV), shown in the format $\Delta E_{dynamical} + \Delta E_{Thomas}$. Theoretical errors may be estimated by allowing each term to vary by $\pm 50\%$. Experimental splittings are given in parentheses below the predictions when they are known.

ℓ	Δm_Λ	Δm_{Λ_c}	Δm_{Λ_b}
$1 \leftrightarrow \frac{3}{2}^- - \frac{1}{2}^-$	$221 - 175$	$75 - 23$	$26 - 3$
	(112 ± 5)	(33 ± 1)	
$2 \leftrightarrow \frac{5}{2}^+ - \frac{3}{2}^+$	$118 - 246$	$39 - 21$	$13 - 3$
	(-60 ± 30)		
$3 \leftrightarrow \frac{7}{2}^- - \frac{5}{2}^-$	$81 - 266$	$26 - 21$	$9 - 3$
	(-10 ± 30)		
$4 \leftrightarrow \frac{9}{2}^+ - \frac{7}{2}^+$	$42 - 199$	$19 - 23$	$7 - 3$

IV. CONCLUSIONS

Figure 1(b) suggests that the tower of Λ_{Q^*} baryons I have described here will provide the simplest setting in which to begin to understand the baryon spin-orbit problem since, at least naively, they ought to have meson-like dynamics. I have shown here that this expectation is in fact correct and that this select tower of baryons has no "spin-orbit puzzle". The key to the solution of the puzzle for this select group of baryons is that for them the microscopic three-body spin-orbit forces described in the literature conspire to produce only meson-like quasi-two-body spin-orbit forces. As a result these states experience the same strong cancellation between dynamical spin-orbit forces and Thomas precession which leads to small spin-orbit effects in mesons.

It is possible that this work has no implications beyond identifying these special states. However, in studying these states we have also developed a new perspective on the forces in baryons which may have a wider utility in confronting the baryon spin-orbit puzzle. In particular, we have seen that the standard classification in the literature of spin-orbit forces as being two- and three-body is somewhat misleading. This classification is correct at the microscopic level (see Eq. (42)), but not very relevant to the issue of whether the spin-orbit forces on a given quark Q appear to arise from the baryon center-of-mass, i.e., whether they are quasi-two-body in character. With the spin-orbit puzzle solved for a slice through the baryon spectrum, it would be odd if there were an intractable spin-orbit problem in baryons. At the least, one may hope that the simple interpretation described here in terms of quasi-two-body (i.e., meson-like) dynamics will lead to further insights into the spin-orbit puzzle.

In fact the arguments presented here for the Λ_{Q^*} baryons are immediately generalizable to the spin-orbit force on the third quark Q in $^2\Sigma_{Q^*}$ and $^4\Sigma_{Q^*}$ states, the total quark spin $1/2$ and $3/2$ Σ_Q baryons with the same spatial wave functions as $\Lambda_{Q^*} \equiv {}^2\Lambda_{Q^*}$. Since these three "uds basis" states [12,1] are sufficient to completely describe the SU(3) baryon spectrum, it might seem that an even more substantial generalization is in hand: in the SU(3) limit,

baryon wave functions have permutation symmetry, so proving that quark 3 has meson-like spin-orbit forces is sufficient to prove that each quark does. However, in the SU(3) limit $\omega_\rho = \omega_\lambda$ and our proofs are inapplicable: with such a degeneracy, leading order spin-orbit effects can occur *via* mixing between $\vec{\rho}$ and $\vec{\lambda}$ excitations. Thus extending the considerations presented here beyond the spin-orbit force on Q in ${}^2\Lambda_{Q^*}$, ${}^2\Sigma_{Q^*}$, and ${}^4\Sigma_{Q^*}$ will be nontrivial. Using our new perspective of quasi-two-body forces, we can visualize at least one obstacle to such extensions. Consider the counterparts to our Λ_{Q^*} state in which the $\vec{\rho}$ variable is in a state with $\ell_\rho \neq 0$ but $\ell_\lambda = 0$. In these circumstances the ud pair will develop an orbital color magnetic moment proportional to $\vec{L}_\rho$ with which the color magnetic moment $\vec{S}_Q/m_Q$ will in general interact.

It would thus seem to require a careful reanalysis of the effects of spin-orbit splittings on the baryon spectrum from this new perspective to determine if all of the *observed* nearly degenerate spin-orbit multiplets can be accommodated within it, and, if they cannot be, to define the next steps along the path to solving this old problem. It of course remains possible, despite the tantalizing fact that mesons and now the Λ_{Q^*} states admit a nonrelativistic solution to their spin-orbit splittings, that the ultimate solution is as suggested in Ref. [7]: that relativistic effects have simply produced a gross enhancement of spin-spin over spin-orbit interactions.

ACKNOWLEDGEMENTS

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